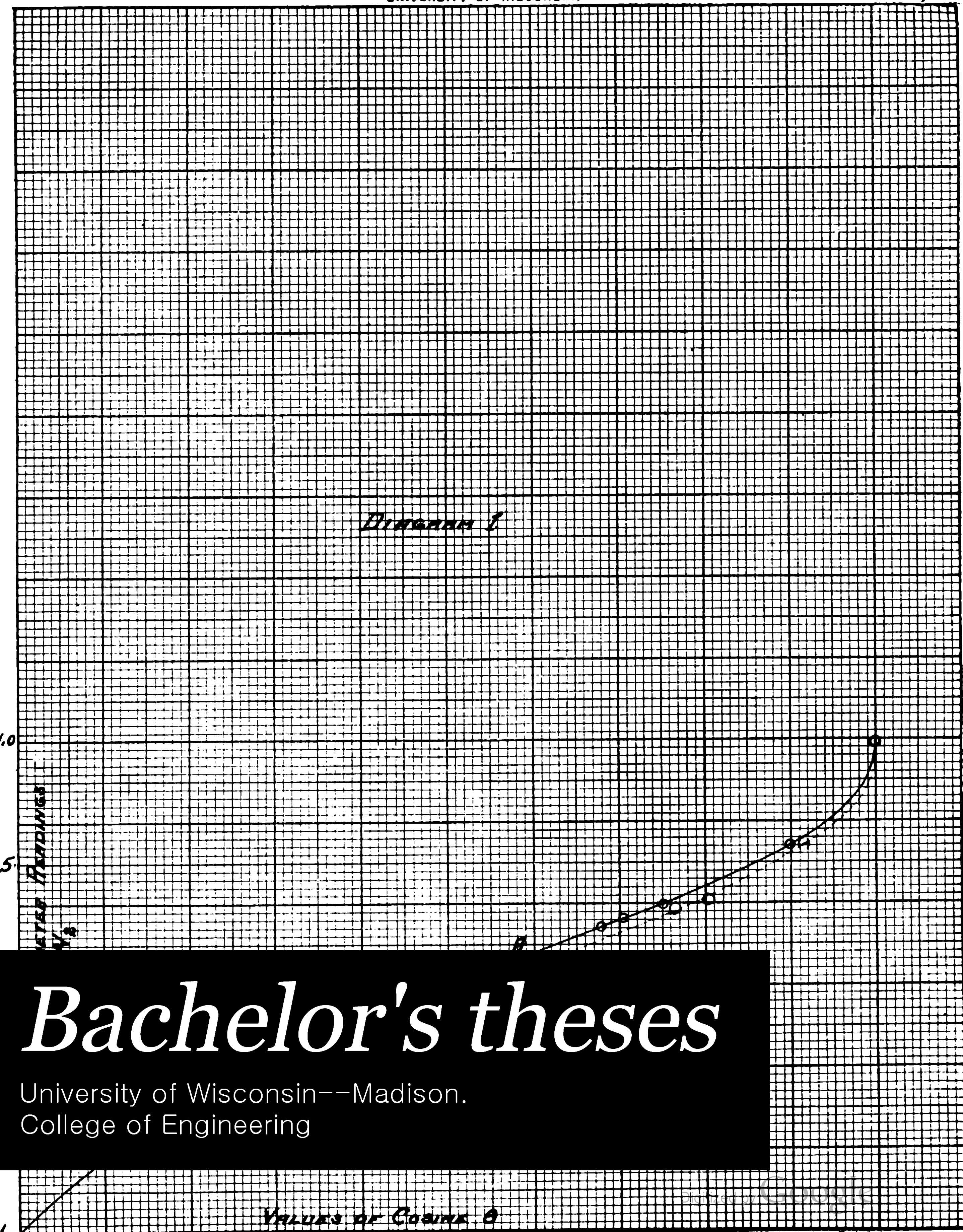

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POWER FACTOR INDICATORS

BY

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and

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INDEX.

POWER FACTOR INDICATORS.

THE THEORY OF POWER FACTOR.

The question of the phase relation of current and pressure in alternating current circuits has been a source of great perplexity to the student of alternating current phenomena and to the designer of alternating current apparatus. Until quite recently, little was known of these relations as is shown by the fact that almost all of the literature on the subject has appeared within the last ten or fifteen years. As further evidence it might be stated that in 1887 William Stanley, Jr., read a paper before the American Institute of Electrical Engineers on "The Phenomena of Retardation in the Induction Coil," which clearly brought out the fact that men, who to-day are foremost among the electrical engineers of the country, did not then know why the product of current times pressure did not give the energy used in an alternating current circuit containing self-induction, the same as in a direct current circuit.

Almost all that has been written on the subject has

been upon the assumption of sine waves of current and pressure. The application of Fourier's Theorem[1] to variable electric current curves has thrown much light on the conditions which exist in case of non-sine waves, still, when non-sine waves are considered, the calculations become so involved and difficult that it has been customary to assume sine waves in most ordinary cases. Whenever power is measured in an alternating current circuit containing self-induction or capacity, its value is taken to be:-

$$I\ E\ \cos\theta$$

where I and E are the effective values of amperes and volts and θ is the angle of lead or lag between the current and pressure. That is, $\cos\theta$ is assumed to be the factor by which the product of the value of amperes and volts, or "apparent watts," must be multiplied to give the power or "true watts." In other words, $\cos\theta$ is assumed to be the ratio of true watts to apparent watts. In 1892, Professor J. A. Fleming,[2] gave to this term the name "Power Factor." The assumption that $\cos\theta$ is equal to the power factor is only true when the curves of current and pressure are sinus-

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[1]Mascarte and Joubert,- Treatise on Electricity and Magnetism. Volume II.

[2]Fleming, Professor J. A.,- Experimental Researches on Alternating Current Transformers. Journal of the Institute of Electrical Engineers, 1892.

oidal. True power factor may be given by the equation:-

$$P.\ F. = \frac{\int_0 e\ i\ dt}{\int_0 e^2 dt \int_0 i^2 dt}$$

where e, i and t are instantaneous values of pressure, current and time. The time lag of two periodic functions of the same frequency is the interval that elapses between the instants when each passes through its zero value in the positive direction. Now unless the two curves are sinusoidal or, at least, exactly similar, the equation for power factor or $\cos\theta$ will not be equal to unity even if the angle if time lag or lead be zero. If we may use the term "phase difference," as the angle whose cosine is the power factor, then $\cos\theta$ will equal the above equation. The angle of phase difference, then, and the angle of time lag or lead are not the same thing. If θ_0 be the angle of phase difference due to dissimilar curves when the angle of lag is zero, then, when the angle of lag is α:-

$$P.\ F. = \cos\theta_0 \quad \cos\theta \quad \cos\alpha.$$ [1]

Even this is only true when both curves are symmetrical and

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[1]Russel, Alexander,- The Theory of Power Factor. London Electrician, November 3rd, 1899.

one is a sine curve. Thus we see the difficulty in trying to define power factor in terms of the angle of lead or lag, especially when both are unsymmetrical and non-sinusoidal.

If we measure power factor by means of a wattmeter, voltmeter and ammeter, giving:-

$$\cos \theta = \frac{\text{true watts}}{\text{apparent watts}}$$

is generally accepted as the angle of lead or lag of the equivalent sine waves.[1] This does not necessarily mean that there is a "wattless" component of power, for a little consideration of the relations of the current and pressure curves to the curves of power will show that there can only be a wattless component of power when the original current and pressure waves are out of phase. That is, as long as the current and pressure curves, no matter how distorted from the sine form, are in phase, the entire power loop will be positive so that there can be no negative or wattless power. At the same time, with non-sine waves, I E is not equal to the true power as $\cos \theta$ or power factor is not unity even though there be no angle of lead or lag. What, then, is the meaning of power factor under such conditions? It may still be considered as the cosine of the

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[1]Jackson, D. C.,- Alternating Currents and Alternating Machinery.

phase angle between equivalent sine waves but perhaps a better way to look at it is to consider the power factor, when there is no phase angle, as the per cent. of the power of a circuit with sine waves, which would be developed by another circuit having non-sine waves but having the same effective values of current and pressure and the same frequency as the former. When there is any angle of lead or lag the power factor becomes a composite quantity whose value depends upon the shape of the curves and the angle of lead or lag, as has already been explained. At present there seems to be little literature on this very important point. If some one would come forward with a mathematical explanation and proof of this, it would be a valuable contribution to the science.

From what has gone before, it is readily seen that the assumption, that the cosine of the angle of lead or lag is the power factor, must be received with considerable caution. All power factor indicators, so called, which have been devised up to the present time, depend for their working upon some function of the angle of lead or lag and do not, therefore, always give true power factors. Then the only way which is left for determining true power factor is the old method of the ratio of wattmeter reading to the product of the voltmeter and ammeter readings.

Of late years the demand for power factor indicators has arisen out of a desire for some means of readily determining the relative amount of wattless current on power lines, operating alternating current motors, with a view of reducing the same. The operator of a power plant cares little for the true power factor of his lines as affected by wave form. What he desires to know is the relation of current and pressure as affecting the regulation and operation of his machines.

For this purpose a number of commercial power factor indicators have been designed. There are, however, a large number of different schemes for determining so-called power factors and some attempt will be made to describe these various methods. They may be classified in general under two heads:-

1. Determination by calculation from readings of different instruments.
2. Determination, directly or indirectly, from the reading of one instrument.

Of the former there are principally:-

(a) The ratio of wattmeter reading to product of voltmeter and ammeter reading, giving true power factor.

(b) The application of the three voltmeter method.

(c) The application of the three ammeter method.

(d) The split dynamometer method.

(e) The two wattmeter method.

These methods are all too well known to need description here. The last method will be discussed under power factor indicators using this principle.

Of the power factor indicators used in the second general class there are quite a variety. They may be classified in the following manner:-

(a) Electrolytic, depending upon electrolysis for a record.

(b) Electromagnetic, using the force of attraction of electromagnets.

(c) Induction, using the effect of a rotating magnetic field upon a disk.

(d) Electrodynamic, utilizing the reaction between coils carrying currents.

Besides these there are a number of oscillographs and curve tracers by means of which the phase relations of current and pressure can be found.

PHASEMETER OF JANET:- The apparatus consists of two styli of iron, s and s', (Fig. 1) which rest upon the exterior surface of a drum covered with a sheet of paper soaked

with a solution of potassium ferro-cyanide and ammonium acetate.[1] The whole cylinder or drum turns in displacing itself laterally. One stylus is connected so as to have a difference of potential between it and the drum, which is in phase with the current, and the other stylus so as to have a potential difference between it and the drum, which is in phase with the E.M.F.

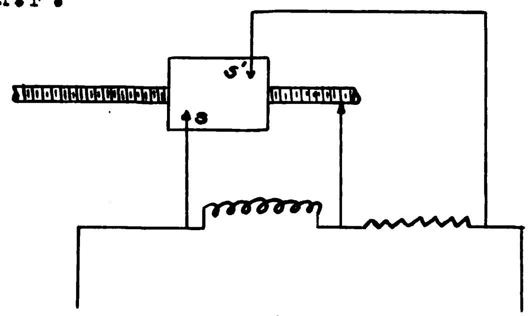

Fig. 1.

When the difference of potential between the styli and the cylinder exceeds a certain positive value, it produces a blue trace upon the paper due to electrolysis of the ferro-cyanide. ABCDE and LMNR (Fig. 2) are the sinusoids drawn as the functions of the values $\frac{2\pi}{T}t$ and the differences of potential between the styli and cylinder.

AA' is the value of the difference of potential necessary for electrolysis. During the interval BCD and MNP

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[1]Fontaine, F.,- Mesure des Differences de Phase. In Bulletine de l'Association des Ingenieurs Electriciens, November, 1899.

electrolysis takes place and two blue traces are drawn upon the sheet of paper. The distance between the centers of the curves (Fig. 2) represents the phase angle + 180°.

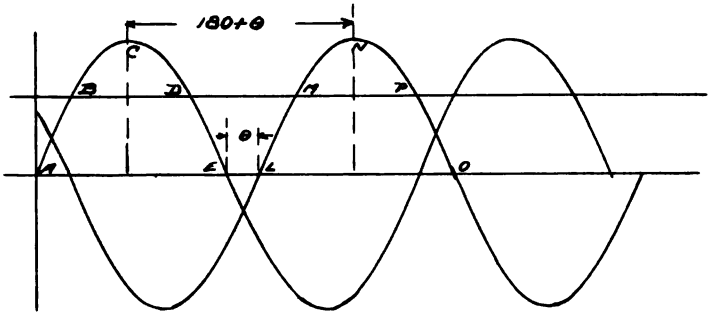

Fig. 2.

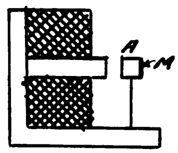

Fig. 3.

PULUY'S PHASEMETER:—[1] Two sinusoidal currents of the same frequency are used to excite two electromagnets, one of which is shown in Figure 3. Opposite one end of the cores of each of the electro-magnets an iron armature, A, is placed. Upon the face of the armature, farthest from the core of the magnet is fixed a small mirror M. If we assume that the magnetization reached by the cores is small, that the hysteresis is negligible and that the vibrations of the armature are of small amplitude we may then also assume that the flux in the core follows the sine law of the current. If, however, the inertia of the armature is also small, the displacements of the latter from its position of equilibrium are at every instant proportional to the flux, that is to say, the current.

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[1]Fontaine, F.,– Mesure des Differences de Phase.

The two armatures (Fig. 4) vibrate in planes normal to each other. A pencil of light concentrated by the aid of a lens, L, is reflected successively upon the mirrors M and M' and upon a screen at E. If the mirror M vibrates alone the point of light reflected upon the screen describes a vertical line, and if M' vibrates alone the ray of light describes a horizontal line. Now, if both vibrate together, the ray of light is at every instant changing direction with a velocity which is the resultant of two others at right angles to each other. Two harmonic motions acting at right angles and having the same frequency and amplitude, but a difference of phase of 90°, will produce, if acting at the same time, a uniform circular motion. If the amplitudes are not the same the result is an ellipse, the major and minor axes of which are the respective paths of the two harmonic motions acting alone.

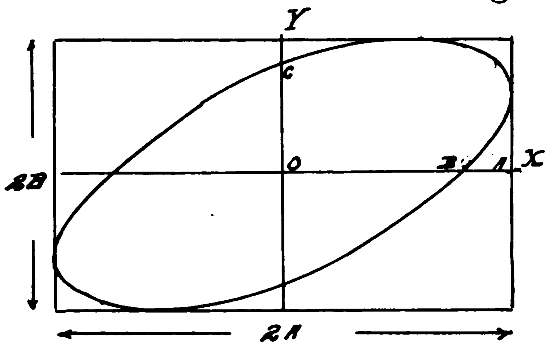

Fig. 5.

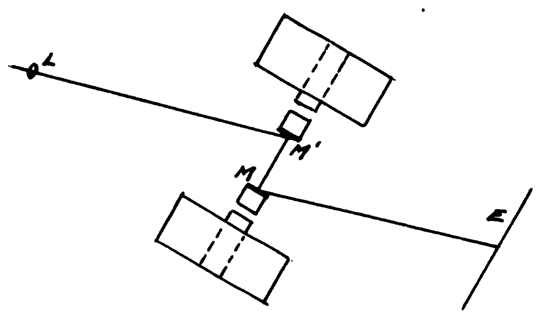

Fig. 4.

The currents in the two electro magnets are represented by the equations:-

$$i_1 = I_1 \sin \omega t$$

$$i_2 = I_2 \sin (\omega t - \theta).$$

The coördinates of the point of light at any instant, k being a constant, are:-

$$X = k\, I_1 \sin \omega t \;= A \sin \; \omega t \;\ldots\ldots\ldots\ldots\ldots(1)$$

$$Y = k\, I_2 \sin (\omega t - \theta) \;= B \sin (\omega t - \theta) \;\ldots\ldots\ldots\ldots(2)$$

Eliminating t from the two equations:-

$$Y = B \sin \; \omega t \cos \theta - B \cos \omega t \sin \theta$$

From (1):-

$$\sin \; t \;=\; \frac{X}{A},$$

and by continuation:-

$$\cos \omega t \;=\; \sqrt{1 - \frac{X^2}{A^2}}\;.$$

Then:-

$$Y = B \;\frac{X}{A}\; \cos \theta - B \;\sqrt{1 - \frac{X^2}{A^2}}\; \sin \theta$$

The equation finally reduces to:-

$$A^2Y^2 - 2\, B^2X^2Y^2 \cos \theta + \; B^2X^2 \;= A^2B^2 \sin^2 \theta$$

This is the equation of an ellipse displaced as shown in Figure (5).

This ellipse is inscribed in a rectangle, which has a

length 2A for a base and 2B for an altitude.

The angular coefficient of the tangent at any point whatever is:-

$$\frac{dY}{dX} = \frac{AB \cos\theta\, Y - B^2X}{A^2Y - A\,B \cos\theta\, X}$$

The tangent to the highest point is determined by the condition:-

$$\frac{dY}{dX} = 0,$$

or,

$$A\,B \cos\theta\, Y - B^2X = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4).$$

Substituting for X its value taken from (4) in equation (3), we have

$$Y = \pm B.$$

It can be proven in the same manner that the tangents to the ellipse parallel to the Y axis are distant 2A from each other.

Rewriting equations (1) and (2):-

$$X = A \sin \omega t \;\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$Y = B \sin(\omega t - \theta) \ldots\ldots\ldots\ldots (2)$$

we have when $X = 0$, $\omega t = N\pi$ where N is any whole number.

$$Y = B \sin(N\pi - \theta) = \pm B \sin\theta = \pm 0\,C,$$

When $Y = 0$:-

$$\omega t - \theta = N\pi.$$

Hence:-

$$\frac{0\,B}{0\,A} = \frac{0\,C}{0\,D} = \sin\theta \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5).$$

Mr. Puluy places upon the screen two scales at right angles to each other and graduated to read from their intersection. He places the screen so that the ellipse cuts the two scales at equal distances from the center. The phase angle is then deduced directly by means of equation (5).

MORELAND'S PHASEMETER:-[1] The scheme devised by S. T. Moreland consists of a divided circuit having in each branch a conductor arranged to vibrate freely. These conductors are stretched at right angles to each other between the poles of strong magnets. The tension of each is adjusted by delicate means until its period of vibration is the same as that of the alternating currents traversing it.

Small mirrors are placed at one end of each vibrating conductor in such a manner that a beam of light is reflected successively upon the two mirrors. If one conductor is made to vibrate alone the ray of light appears in the form of a band of light in one plane, and if the other vibrates alone another band of light appears at right angles to the first. When both conductors vibrate together a band of light appears half way between, or diagonal, to the other

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[1]Moreland, S. T.,- In Electrical Engineer, XXVI:237, September 28, 1893.

two positions, if the currents in each coil are in phase. If the currents are out of phase the ray of light describes an ellipse. This scheme is very similar to the one credited to Puluy.

CLAUDE'S POWER FACTOR METER:-[1] Claude has worked out a power factor meter which belongs in the same general class with the Puluy instrument. However, instead of using two armatures, he places the two electro-magnets in direct opposition and causes them to actuate the same armature.

It has been proven that:-

$$\frac{D_2}{D_1} = \cos\frac{\theta}{2},$$

where D_2 and D_1 are the lengths of the paths of light when there is a phase displacement between the currents in the two coils, and when there is no phase displacement.

Mr. Browne endeavored to show, in the first part of his article on power factor indicators, that all deductions of power factor, based on cosine and tangent functions of the phase angle are not satisfactory when θ approaches zero. According to this, Claude's method could not be depended upon to give reliable results when the phase angle θ approaches zero.

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[1]Browne,- in Report of American Institute of Electrical Engineers, 1901, p. 294.

RAYLEIGH PHASE METER:-[1] Lord Rayleigh devised a phase meter which is quite different from any before described. It is somewhat similar to a galvanometer except that it has a soft iron needle in place of a magnet. This needle is suspended by a fine torsion fiber and carries a mirror in the usual manner for galvanometers, the needle being inclined 45° to the direction of the magnetic force. This force is due to the currents in two coils, the common axis of the coils being horizontal and passing through the center of the needle. The coils are placed on opposite sides and at a distance from the needle which can be varied at will. A plan of the arrangement is shown in Figure 6, where C C and P P are the two coils, the common axis A B passing through the needle N.

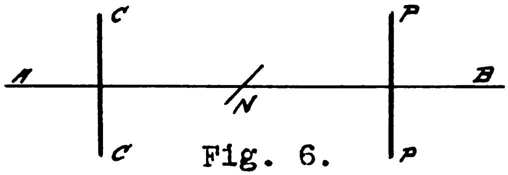

Fig. 6.

If the currents in the coils are of simple type, the magnetic force along A B may be represented by:-

$$A \cos \omega t \quad \text{and} \quad B \cos \omega t - \theta,$$

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[1]Rayleigh, J. W. S.,- Instrument for Phase Angle Measurement. London Electrician, June 4th, 1897.

where θ is the angle of phase displacement. If either force act alone, the deflecting couple is represented by A^2 or B^2 but if the two couples coöperate, the corresponding effect is:-

$$C^2 = A^2 \pm B^2 + 2\,AB \cos\theta,$$

reducing to $C^2 = A^2 \pm B^2$ only when θ is zero, or 180°.

The method consists of measuring, by means of a common telescope and scale, the deflections which, for small values, are proportional to A, B, and C.

From these, the angle θ can be readily deduced. The determination of phase difference between current and pressure can thus be made independent of any measurements of their respective values.

FERRARIS PHASEMETER:-[1] This meter consists of two fixed coils A and B (Fig. 7) at right angles to each other and a movable coil C. The coils A and B carry the currents under consideration. In the space between the two coils the third coil, C, is placed, free to turn about on its axis at the intersection of

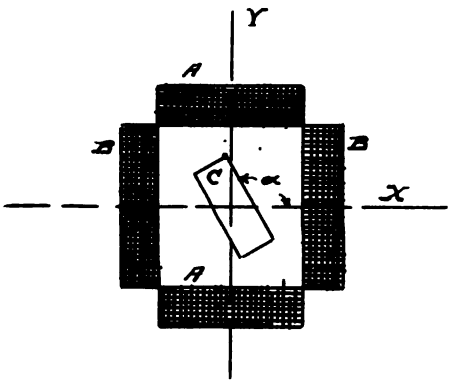

Fig. 7.

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[1]Fontaine, F.,- Mesure des Differences de Phase.

the X and Y coördinates. The positions of this coil are represented upon a graduated scale.

A rotating field whose intensity varies as the vectors of an ellipse, is set up in the space which separates the two coils. The components of the field,

$$X = A \sin \omega t,$$

$$Y = B \sin (\omega t - \theta)$$

are perpendicular.

We then proceed to show that the movement of the coil C gives a deflection in simple relation with the vector representing the field in the plane of the coil C.

The flux in the coil, C, produced by the coil A is:-

$$N' = A \sin \omega t \sin \alpha;$$

the E. M. F., induced:-

$$E' = \frac{D N'}{D t} = A \omega \cos \omega t \sin \alpha.$$

In a similar manner, the flux in the coil C, produced by the coil B, is expressed by:-

$$N'' = - B \sin (\omega t - \theta) \cos \alpha,$$

and the E. M. F., induced, is expressed by the equation:-

$$E'' = - \frac{D N''}{D t} = -B \omega \cos(\omega t - \theta) \cos \alpha.$$

The total induced E. M. F. at any instant t is given by the equation:-

$$E = - \omega [A \cos \omega t \sin \alpha + B \cos (\omega t - \theta) \cos\alpha].$$

It follows that the maximum induced E. M. F. is the resultant of the E. M. F.'S $- \omega A \sin$ and $- \omega B \cos \alpha$, displaced by the angle θ (Fig. 8).

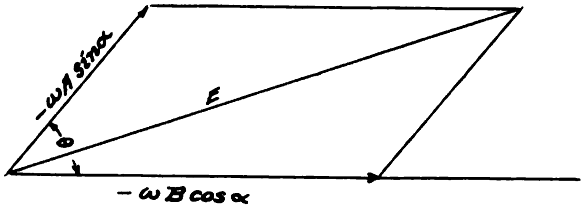

Fig. 8.

Representing the resultant by E, from Fig. 8, we have:-

$$E^2 - \omega^2(A^2 \sin^2\alpha + B^2 \cos^2\alpha - 2 AB \sin \alpha \cos \alpha \cos \theta).$$

The equation of the ellipse is:-

$$A^2Y^2 - 2 ABXY \cos \theta + B^2 X^2 = A^2B^2\sin^2 \theta \quad ....(6).$$

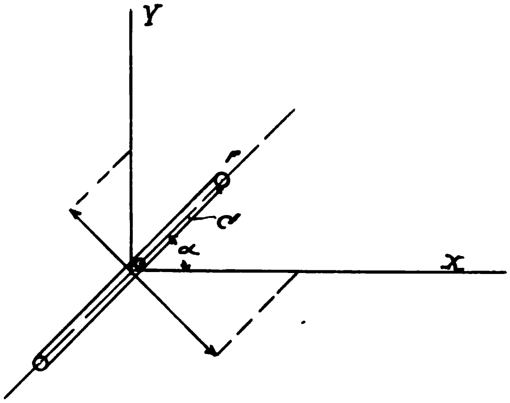

Fig. 9.

The line $O\,F$ has the equation:-

$$Y = X\ tg\alpha,$$

and the square of the vector of the ellipse in this direction is represented by:-

$$X^2 + Y^2 = X^2 + X^2 tg^2\alpha = \frac{X^2}{\cos^2\alpha}.$$

Putting Y equal to $X\ tg\alpha$ in equation (6), we have:-

$$A^2X^2tg^2\alpha - 2\ AB\ \cos\theta\ X^2\ tg\alpha + B^2X^2 = A^2B^2\sin^2\theta \ \ldots\ldots\ (7).$$

$$X^2 = \frac{A^2B^2\sin^2\theta}{A^2\sin^2\alpha - 2AB\ \cos\theta\ \sin\alpha\ \cos\alpha + B^2\cos^2\alpha} \qquad \frac{A^2B^2\sin^2\theta}{E^2}\,\omega^2$$

Then the vector is equal to:-

$$\frac{A\ B\ \sin\theta}{E}\,\omega.$$

Let δ represent the torsion of the electrodynamometer and KA the constant, then:-

$$K\delta = E^2, \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (8)$$

which gives for the vector:-

$$\frac{A\ B\ \sin\theta}{\sqrt{K\ \delta}} = \frac{K'}{\sqrt{\delta}}. \ \ldots\ldots\ldots\ldots\ldots\ldots\ (9)$$

This last equation permits us to trace the ellipse by points. Having drawn the ellipse, the phase angle can be determined as previously shown.

ENGELMEYER METER:-[1] This meter consists of two electro-magnets traversed by the currents considered. One of them, B, is movable about the point O, the intersection of their axes.

The position of B is represented on a graduated disk S (Fig. 10). The point θ is occupied by a vibrating armature furnished with a mirror M, which reflects a ray of light upon the screen E. One can easily show that if the fluxes produced at O by the two electro-magnets are equal the ray of light will trace a circle if the coils are inclined at an angle equal to the supplement of the phase angle.

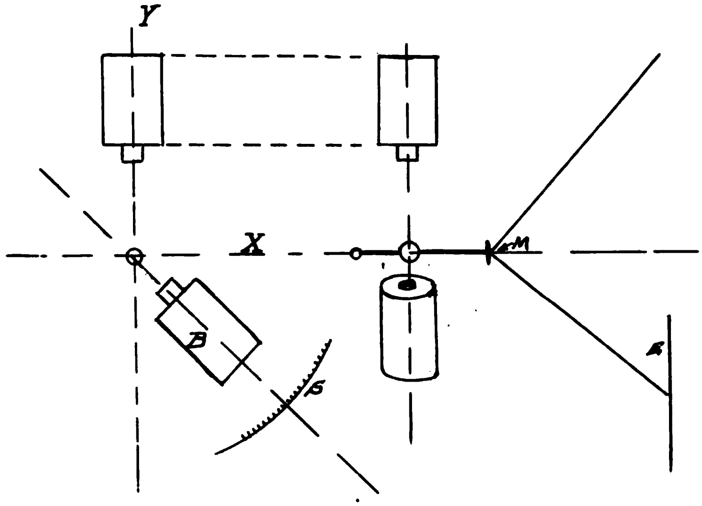

Fig. 10.

Taking for the axis of Y the axis of the field coil A, and an axis perpendicular to it for the X axis, the projections of the field components upon these axes are:-

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[1]Fontaine, F.,- Mesure des Differences de Phase.

$X = A \sin \omega t \sin \theta$, and $Y = A \sin (\omega t - \theta) - A \sin \omega t \cos \theta$

$$\sqrt{X^2 Y^2} = \sqrt{A^2 \sin^2 \omega t \sin^2 \theta + A^2 \cos^2 \omega t \sin^2 \theta} = A \sin \theta$$

The angle of phase is shown directly on the scale.

KORDA METER:-[1] If the two coils of a Ferraris inductor, traversed by currents of the same frequency, are inclined at an angle equal to the supplement of the phase angle, the preceding relation still exists for the two fields are also inclined at $180° - \theta$, since they are perpendicular to the coils.

The coil, C, will be the seat of an induced E. M. F., due to the variability of the intensity of the rotating field and to the variations in the velocity of the flux during turning. The first is easily seen, and as to the second part of the preceding statement, we have, referring to the components of the field:-

$$X = A \sin \omega t,$$

$$Y = B \sin (\omega t - \theta).$$

The tangent of the angle α in the direction of the resultant flux with the axis of X at any instant, t, is:-

$$\frac{Y}{X} = tg\omega = \frac{B}{A} \left(\cos \theta - \frac{\sin \theta}{tg\ at}\right) \quad \ldots.(10)$$

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[1]Fontaine, F.,- Mesure des Differences de Phase.

Differentiating:-

$$\frac{1}{\cos^2\omega}\cdot\frac{d\omega}{dt} = \frac{B}{A}\,\frac{\sin\theta\,\dfrac{a}{\cos^2\omega t}}{tg^2\,a\,t} = \frac{B}{A}\sin\theta\cdot\frac{A}{\sin^2 at}$$

or:-

$$\frac{d\omega}{dt} = \frac{B}{A}\sin\theta\,\frac{A}{\sin^2 at}\cos^2\omega.$$

Substituting for ω its value, taken from equation (10):-

$$\frac{d\omega}{dt} = \frac{B}{A}\sin\theta\,\frac{a}{\sin^2 at}\cdot\frac{1}{\left[1+\dfrac{B^2}{A^2}\cos\theta-\dfrac{\sin\theta}{tg\,at}\right]^2}$$

It is thus seen that the angular velocity depends upon the time, if $A = B$.

Imagine, on the contrary, a rotating field:-

$$\frac{Y}{X} = tg\omega = -\,tg\,at$$

$$\omega = at + 180°.$$

$$\frac{d\omega}{at} = a = \text{constant}.$$

When in the case of a rotating field the velocity of the coil C relative to the field is zero at each instant, the Korda meter is easily used.

PHASEMETER OF HESS:-[1] Two currents of the same ampli-

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[1]Fontaine, F.,- Mesure des Differences de Phase.

tude, whose phase difference is supposed to be known, are made to pass through the coils of an instrument of the Ferraris type. Then, by inclining the coils by an angle equal to the supplement of the phase angle, we obtain a rotating field of constant intensity. The components of this field at any instant t are:-

$$X = A \sin at \sin \theta.$$

$$Y = A \sin (at - \theta) - A \sin at \cos\theta$$

$$= + A \cos at \sin \theta.$$

The tangent of the angle of the resultant field at this instant with the axis of Y is:-

$$tg\alpha = \frac{X}{Y} = - tg\ at$$

$$\alpha = 180° + at.$$

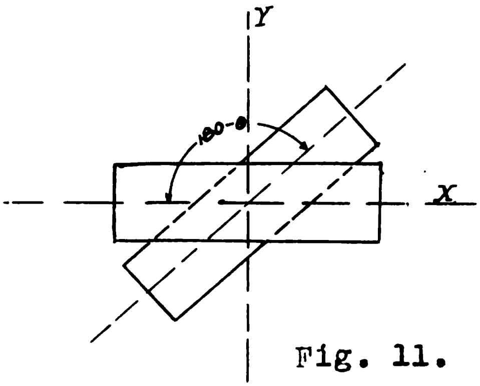

Fig. 11.

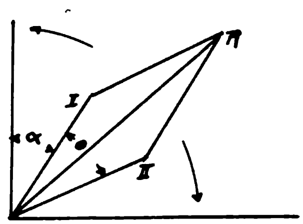

Fig. 12.

Suppose that one superimposes upon the first set of coils a second similar set, traversed by two currents of like amplitude as the preceding but displaced from them by an angle θ. They will also produce a rotating field of con-

stant strength.

The components at any instant t are:-

$$X = A \sin (at - \theta) \sin \theta;$$

$$Y = A \cos (at - \theta) \sin \theta;$$

From which one has:-

$$tg\ \alpha' = - tg\ (at - \theta),$$

$$\alpha' = 180° + at - \theta.$$

The resultant of these two equal fields is evidently in the same direction as the bisector of the angle θ. One can always adjust the currents in the coils in such a manner that the rotating fields move in the opposite directions. As their velocity is the same their resultant lies in the same direction. It is upon this principle that the Hess phasemeter is based. The coils belonging to the same set have the same impedance (because the derived currents are equal) but the inductance is different. It is this condition which necessitates the angle of inclination $180° - \theta$.

The coils of the second set are identical with the first set of coils. In the interior of the apparatus is suspended a soft iron needle intended to indicate the constant direction of the resultant field.

ARNO'S PHASEMETER:-[1] This phasemeter does not depend upon the condition of equal amplitudes of current which was essential in the preceding apparatus.

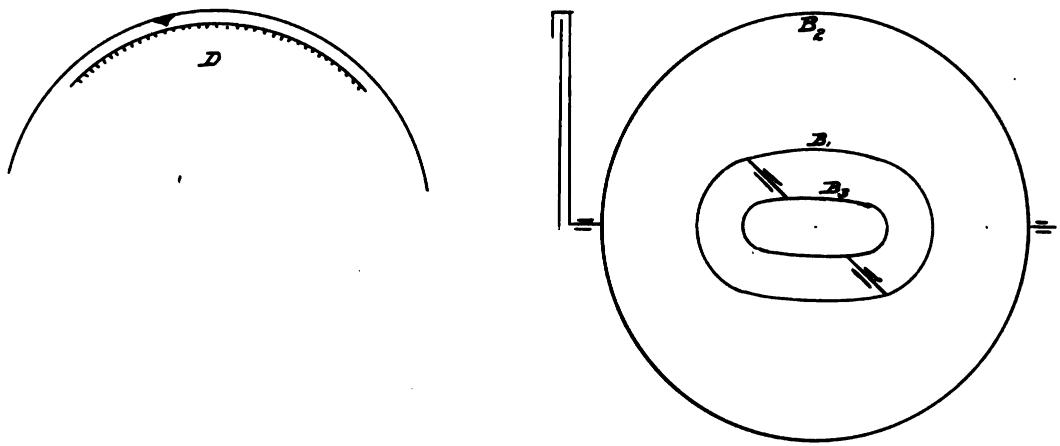

Fig. 13.

It consists of an inductor of the Ferraris type made up of two coils intersecting each other along a diameter of one, and a third coil, B_3, whose axis is a diameter of B_1.

The coils B_2 and B_3 can be turned about a horizontal axis. Their positions are represented on a graduated scale D. B_1 can turn about a vertical axis passing through its center. The coil, B_3, is acted upon by coils B_1 and B_2 which are traversed by the currents whose phase difference is desired. One has the following conditions to start with. B_2 is placed vertical and B_1 and B_3 are horizon-

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[1]Fontaine, F.,- Mesure des Differences de Phase.

tal. The axis of B_3 is brought into the plane of B_1 and the deflection δ given by the electrodynamometer noted. B is then turned until it occupies a vertical position and the deflection δ' observed.

If the amplitudes of the two currents are not the same we have the condition $\delta' \gtrless \delta$. But one can always make $\delta' > \delta$ because it suffices in the contrary case to change the currents in the two coils. This having been done, B remains vertical and the system of coils B_1 and B_3 is made to turn until the deviation is equal to δ. α is then the angle made by the planes of the coils B_1 and B_3.

When B_3 was in the plane of B_1 and B_2 was vertical this latter was without action upon B_3. If:-

$$i_1 = I_1 \sin at,$$

is the current in B_1 and M is the coefficient of mutual induction between B_1 and B_3, B_3 is the seat of an in-

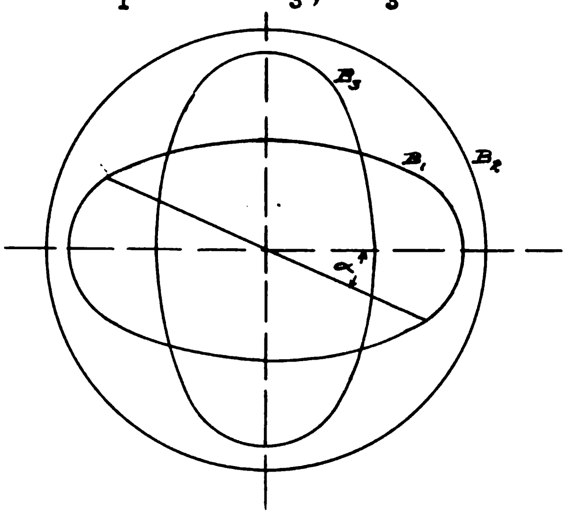

Fig. 14.

duced e. m. f. as follows:-

$$e = - \frac{di}{dt} = - M\, a . I . \cos at .$$

Its maximum is:-

$$E_1 = - MaI_1 = \sqrt{K\delta} \ldots\ldots\ldots\ldots\ldots\ldots (A).$$

In the second case when the coils B and B were at an angle, α, the coefficient of mutual induction is reduced to $M \cos \alpha$ and B_3 is the seat of an induced e. m. f., whose maximum is:-

$$E_2 = -MaI \cos \alpha = \sqrt{K\delta} \ldots\ldots\ldots (B).$$

Equations (A) and (B) give:-

$$I = I \cos \alpha .$$

The two unequal currents acting upon the coil B_3 have the same effect by this disposition. It now suffices to incline the coil B_2 upon the vertical at such an angle that for all the positions of B_3 the deviation of the electrodynamometer remains constant. The ellipse is then transformed into a circle and the inclination to the vertical is represented upon the disk, D, so graduated as to read the phase angle directly.

Angelmeyer, Korda, Hess, Rossi,[1] and Arno make use of the principle that if two coils carrying equal currents are inclined at an angle equal to the supplement of the angle between the currents considered, the resulting rotating field will have a constant value. If the inclination of the coils be varied until this condition is obtained, the phase angle is indicated directly.

HOLDEN PHASOMETER:- This three-phase instrument is based on the fact that the current in one line is in quadrature with the E. M. F. between the other two lines. The main current in line A (Fig. 15) is sent through a pair of fixed coils F and F. In this field is suspended a movable system of two coils whose zero position is perpendicular to the series coils.

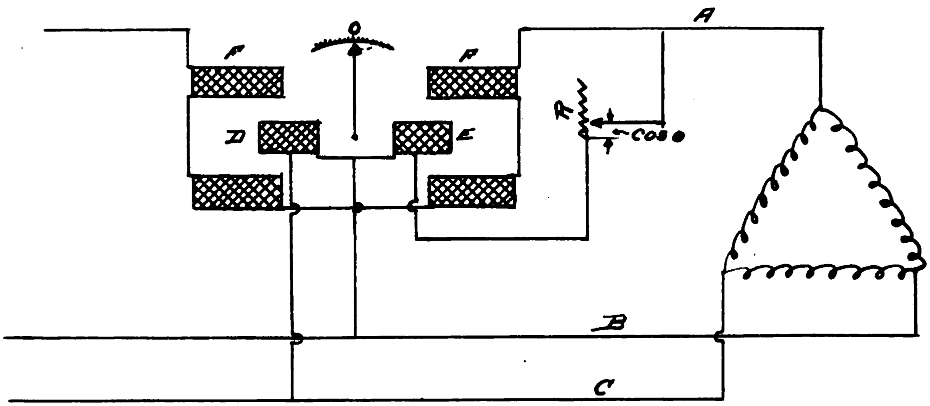

Fig. 15.

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[1]L'Eclairage Electrique, Volume XV, pp. 133, 332 to 335.

One shunt coil D is connected across the lines B and C so that with no phase lag there is no couple acting upon the movable system. The second shunt coil E is connected across lines A and C through an adjustable resistance R. When the current lags the deflection of the movable system is indicated by the needle N. The couple due to the second shunt coil E can be varied by the resistance R and the needle brought back to zero. The resistance needed to accomplish this corresponds to the angle of lag in this particular instance. It is thus seen that by proper calibration of the resistance R the phase angle can be determined directly.

This scheme can be adapted to single phase work by connecting one shunt coil across the lines through a coil of high inductance so as to get a current as near as possible in quadrature with the pressure. Any deflection caused by a phase lag is annulled by shunting the inductance coil by an adjustable non-inductive resistance calibrated to give the phase angle directly. Any effect due to vibrations in frequency may be compensated by connecting a condenser in series with the second shunt coil.

CARCANO POWER FACTOR INDICATOR:- This instrument is very similar to the one above, The apparatus consists of a

disc, D, (Fig. 16) acted upon by a rotating field set up by the currents flowing in the coils A, A, and B. A, A, are the series or current coils and B are the shunt coils.

When the currents in the two sets of coils are out of phase the revolving field produced acts upon the disc, causing it to rotate.

Between the lines 1 and 2, a resistance is inserted of which M is the middle point, and between the lines 2 and 3 another equal resistance is divided into two portions,- a fixed resistance and a variable rheostat,- having a number of contact points. The shunt coils are connected between the point M and the sliding arm of the rheostat. By changing the position of the rheostat arm the current i in the shunt coils can be brought into phase with the main current I flowing in the series coils. When this condition is reached there will be no couple acting upon the disc. Consequently, every point on the rheostat producing equilibrium of the disc corresponds to a given value of the angle θ which represents the angle between the current

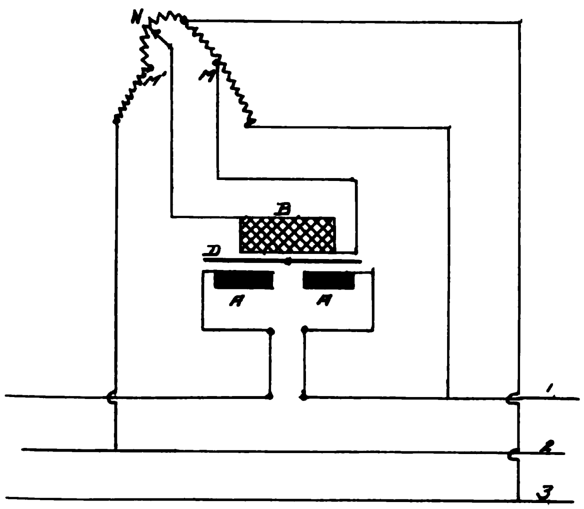

Fig. 16.

and pressure. By a proper calibration of the resistance the phase angle can be read off directly.

This is a zero method and as such possesses some advantages. It does not, however, permit the following of sharp variations in the power factor. By restraining the disc in its zero position by a system of springs, which could be made by the deflection of the disc to close the circuit of a relay recording instrument so constructed that the motion of the recording pointer would change the position of the rheostat until the disc came back to zero, we should have a recording instrument.

Of all the methods, which may be classified under this head, the Dobrowolski method, on which the Conrad meter is based, is the only one which is practical.

DOBROWOLSKI PHASEMETER:- [1] In the ordinary induction wattmeter, the magnetic flux due to the shunt winding is in quadrature with that of the series winding, the result being a rotating or shifting field. If the shunt flux be brought into phase with the series flux, an alternating field only will be set up as long as the current and pressure are in phase. If, however, the current lag, the field will rotate

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[1] Browne,- Power Factor Indicators. A. I. E. E., 1901.

in one direction; if it lead, the field will rotate in the opposite direction. The torque developed on a disc placed in this field is, in either case, proportional to:-

$$E\ I\ \sin\theta,$$

the wattless volt-amperes.

In 1894, Dobrowolski described a phasemeter working on this principle.[1] It consists of two concentric coils at right angles to each other, surrounding a disc which is held at the zero point by a spring when the wattless volt-amperes are zero, but which deflect one way or the other, when the current is out of phase, by an amount proportional to:-

$$E\ I\ \sin\theta.$$

Later, he adapted this meter to three-phase working, by using three current coils 120° apart and one pressure coil. The meter, therefore, would work upon the assumption that the pressure across the three lines are equal and 120° apart.

CONRAD POWER FACTOR INDICATOR:-[2] This meter works on practically the same principle as the Dobrowolski. It has a suitable number of current coils, corresponding to the number of phases, placed apart at suitable angles. A pres-

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[1]London Electrician, September 21st, 1894.
[2]United States Patent, Number 695,913, March 25th, 1902.

sure coil wound on a laminated iron ring or spool is placed in the current coils with its axis perpendicular to the axis of the current coils. In place of a disc, a magnetic vane of soft iron is placed with its axis coincident with the axis of the pressure coil and with wings of the vane outside of the current coils, one above and one below.

ARNO'S TANGENT PHASEMETER:- [1] A second phasemeter of Arno's consists of a Siemens electrodynamometer with an additional pair of coils closed upon themselves and fastened together at right angles. The pair, as a whole, is suspended within the two other coils and may be held in any position by a torsion spring. In (Fig. 17) let A and B be the two dynamometer, and C and D, the pair of short-circuited coils.

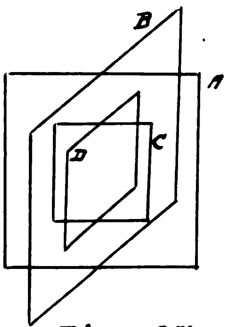

Fig. 17.

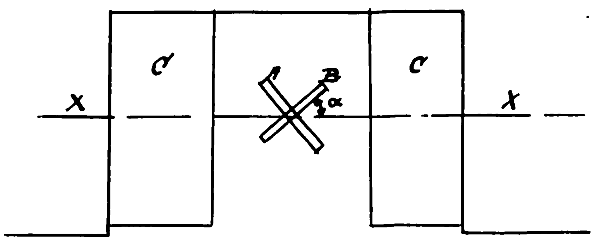

Fig. 18.

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[1]Browne,- Power Factor Indicators, A. I. E. E. 1901.

Passing the two currents through A and B, respectively, we shall get a deflection of the movable coil B of the dynamometer proper. Let d_1 be the torsion angle required to keep B in position. At the same time, the system D C will be deflected, its angle of torsion being d_2. Then it has been proven that:-

$$k \frac{d_2}{d_1} = \tan \theta,$$

where k is the constant of the instrument and θ the phase angle.

TUMA'S PHASEMETER:- [1] Tuma devised a very simple instrument for determining the phase angle directly. Between the two current coils C C is hung a pair of small coils placed at right angles to each other. Through B is passed a current which is in phase with the pressure, and through A, a current which is in quadrature with it. Now, C C being fixed and A B free to turn, Tuma has proven that the angle α which B makes with the axis of the current coils is the phase angle which can thus be determined directly or the scale of the instrument can be so divided as to read cosines.

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[1]Browne,- Power Factor Indicators. A.I.E.E., 1901.

Bowie has applied this scheme to two and three phase circuits by placing the current coils in different phases and obtaining the power factor of the system from this.[1]

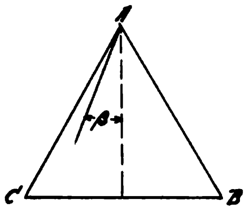

Fig. 19.

For three phase circuits this would give the angle of lag of the current in one leg, namely, the angle by which the current lags behind the bisector of the corresponding vertex of the delta. It is evident, then, that to find the power factor of the system by this method, the system must be perfectly balanced, the pressures must be exactly 120° apart and the waves of current and pressure must be sine waves.

The above scheme is really an application of the two wattmeter method of determining power factor. Power Factor Indicators, based upon this principle, have been placed upon the market by Elliott Brothers of London and by the General Electric Company. These instruments, theoretically, are equivalent to two wattmeters so arranged that the Power Factor can be read off directly upon a properly calibrated scale. The readings of these instruments, therefore, take the place of two separate wattmeter readings and the subse-

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[1]Bowie, A. J.,- Power Factor Indicator. Electrical World and Engineer, October 27th, 1900.

quent calculations necessary for obtaining the Power Factor. The conditions, however, which affect the accuracy of the two wattmeter method apply to these instruments as well, and as they are of considerable importance they will be briefly discussed.

The fact that the ratio of the readings of two wattmeters, used to measure the power of a three-phase system, can be used to represent Power Factor has been known for some time. Mr. Frankenfield wrote an article upon this subject, which was published in the Wisconsin Engineer in 1896. His results were deduced from the assumption of sine waves of current and pressure and balanced loads.

The above conditions are generally assumed in practice, but are not always met with. When the waves of current and pressure are non-sinusoidal the two wattmeter method can not be depended upon to give accurate results. This has been proved quite conclusively by Messrs. Bowie[1] and Block.[2] Both writers deduce the well known tangent formula:-

$$\theta = \sqrt{3}\ \frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\ \frac{1 - R}{1 + R},$$

which applies to balanced loads and sine waves, and then pro-

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[1] The Electrical World and Engineer, December 9th, 1899. page 898.
[2] Elek. Zeitung, December 3rd, 1893.

ceed to show the errors involved when the waves of current and pressure contain the higher harmonics. The results show that the angle deduced from the tangent formula, the angle of time lag or lead, and the angle, whose cosine is the true Power Factor, are three different angles.

The tangent formula for three-phase systems almost always gives higher values for the cosine than are obtained from current pressure and true power measurements. It is entirely possible that, although the cosine computed by the tangent formula for a certain case may equal unity, the true Power Factor may be only 0.3 or 0.5.

In such cases, while the value of the angle obtained from the tangent formula is generally smaller than the actual phase displacement of the fundamental waves, yet, it is nearer to the latter than the still greater angle obtained from the true Power Factor.

In most cases in practice, the two wattmeter method can be depended upon to give fairly accurate results, more accurate, in fact, than the ordinary wattmeter, ammeter and voltmeter methods. It substitutes the ratio of the readings of one instrument for the readings of several different instruments, which is certainly of great importance on account of the errors involved when different instruments are used. It is thus seen that instruments depending upon this

principle are very well adapted to the measurement of Power Factor.

EXPERIMENTAL DATA.

In the light of the foregoing discussion, some experiments were undertaken to determine with what degree of accuracy the two wattmeter method and tangent formula could be used, in actual practice on non-sine waves, to obtain true Power Factor. At the same time it was desired to check up the readings of several power factor indicators depending upon this principle but, as none could be obtained at the time, this project had to be abandoned.

At first, it was thought that a three-phase synchronous motor could be used to obtain different power factors for both leading and lagging currents by over and under exciting its field, but, upon trial, it was found that the hunting of the motor was so great as to make definite or accurate readings impossible. The desired result, so far as lagging currents were concerned, was finally obtained by means of an induction motor, for it is well known that an induction motor, especially any of the earlier makes, has a very low power factor on no load and that the power factor gradually increases up to a maximum, which, for most motors is be-

tween eighty and ninety per cent. To obtain higher values of Power Factor, non-inductive resistances consisting of lamp banks were placed in parallel with the windings of the motor.

The apparatus consisted of a 110 volt, three-phase, Wagner generator, belted to a shunt motor, and a 3 H. P. Allegemeine Electricitate Gesellschaft, three-phase induction motor, belted to a small, direct current generator for a load. The Wagner generator was used because it gave a pressure wave, which varied from the sine wave form to a greater extent than any other obtainable in the laboratory, being an irregular flat top curve as shown in Figure 20.

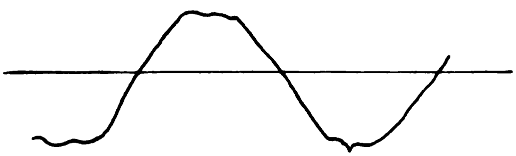


Fig. 20.

By placing a rheostat in the shunt motor field, and one in the field of the three-phase generator, the voltage on the induction motor could easily be adjusted and kept at a constant value. A wattmeter, a voltmeter and ammeter of suitable range were calibrated and used in the test. By

means of a system of switching, as shown in the diagram of connections, (Fig. 21), readings were taken of amperes and watts in each leg and the pressure from each line to the neutral point with one set of instruments. As the pressure and load were kept constant for each set of readings, this gave more accurate results than could be obtained with a number of different instruments. Readings were also taken of total watts by means of the two wattmeter method, the watts in each leg being taken merely for a check. Values of the tan θ'' and cos θ'' were obtained from the two wattmeter readings, using the formula:-

$$\tan \theta'' = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}\,.$$

Values of cos θ' or Power Factor were obtained by dividing the sum of the two wattmeter readings by the sum of the volt-amperes in each leg.

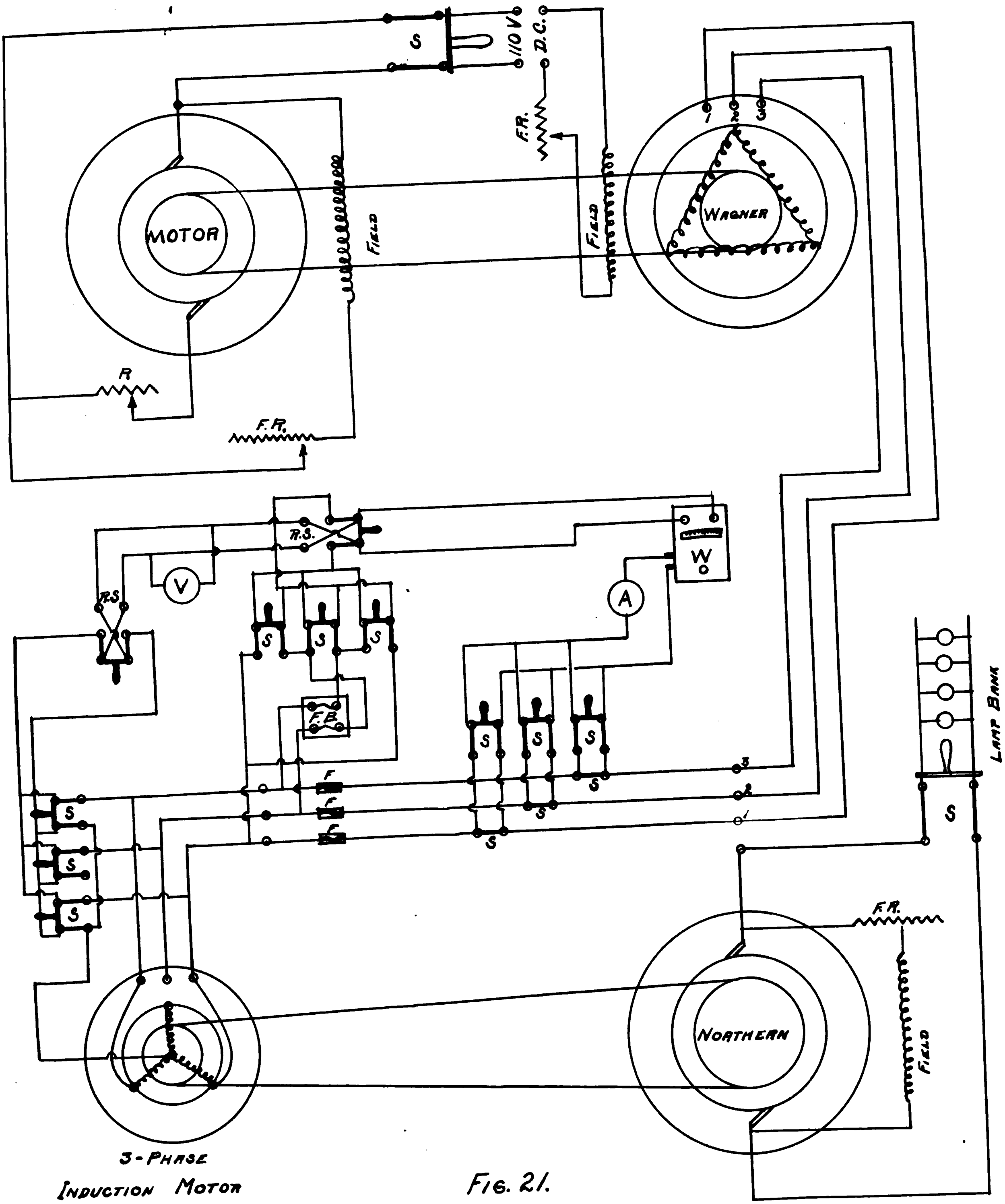

S
110 V
D.C.
F.R.
MOTOR
FIELD
WAGNER
1
2
3
FIELD
R
F.R.
R.S.
V
W
A
R.S.
S
S
S
F.B.
S
S
S
F
F
F
3
2
1
S
S
S
S
S
S
LAMP BANK
S
F.R.
NORTHERN
FIELD
3-PHASE
INDUCTION MOTOR

FIG. 21.

DATA

No	Pressure per Leg			Amperes per Leg			Wattmeter Readings		Total Watts
	1	2	3	1	2	3	W_1	W_2	$W_1 + W_2$
1	57.0	58.1	58.4	13.8	13.8	13.3	480	1410	1890
2	57.2	58.4	58.8	15.8	15.7	15.3	550	1620	2170
3	57.0	58.1	58.4	12.8	12.7	12.3	430	1280	1710
4	57.5	58.5	58.9	19.2	186	18.4	630	1900	2530
5	56.9	58.0	58.2	11.7	11.7	11.2	380	1170	1550
6	57.0	58.0	58.0	9.56	9.5	9.3	280	955	1235
7	57.0	57.8	57.8	8.35	8.5	8.05	225	840	1065
8	57.0	58.0	58.0	7.25	7.3	6.8	170	745	915
9	56.7	57.9	58.0	4.28	4.38	4.1	-65	380	315
10	56.0	57.4	57.4	3.47	3.49	3.39	-135	265	130
11	59.0	60.0	60.4	22.2	22.5	21.7	1310	2310	3620
12	60.9	62.0	61.8	10.9	11.1	11.1	1010	1020	2030

No	Volt Amperes per Leg			Total Volt-Amps	P.F. = Cos θ $\sqrt{3}\,\frac{W_1+W_2}{\sqrt{3}\text{-Amps}}$	Tan θ″ $= \sqrt{3}\,\frac{W_1-W_2}{W_1+W_2}$	Cos θ″	$\frac{W_1}{W_2}$
	1	2	3					
1	787	802	776	2363	.80	.852	.7614	.34
2	905	917	900	2722	.796	.854	.7604	.339
3	727	741	718	2186	.783	.861	.7585	.336
4	1100	1090	1080	3270	.775	.869	.7547	.332
5	665	679	652	1196	.775	.875	.7528	.324
6	545	550	539	1634	.755	.946	.7274	.293
7	476	491	465	1432	.745	1.000	.707	.268
8	413	423	394	1230	.742	1.088	.677	.228
9	243	254	238	735	.428	2.440	.3785	-.171
10	194	200	194	588	.221	5.320	.1847	-.510
11	1300	1350	1310	3960	.914	.478	.9026	+.567
12	664	688	686	2038	.997	.0081	.9999	+.99

CONCLUSIONS.

The results show that in the case of non-sine waves the Power Factor derived from the tangent formula is not equal to the true Power Factor. The curve, A, (Diagram 1) was plotted, using the ratio of wattmeter readings as ordinates and corresponding values of cos θ" as abscissas. Curve B was plotted in the same manner, using values of true Power Factor in the place of the cosine θ" values obtained from the tangent formula. The curves show, as previously stated, that, for a given phase angle corresponding to the ratio of two wattmeter readings, the tangent formula gives smaller values for cos θ than are obtained from the true Power Factor.

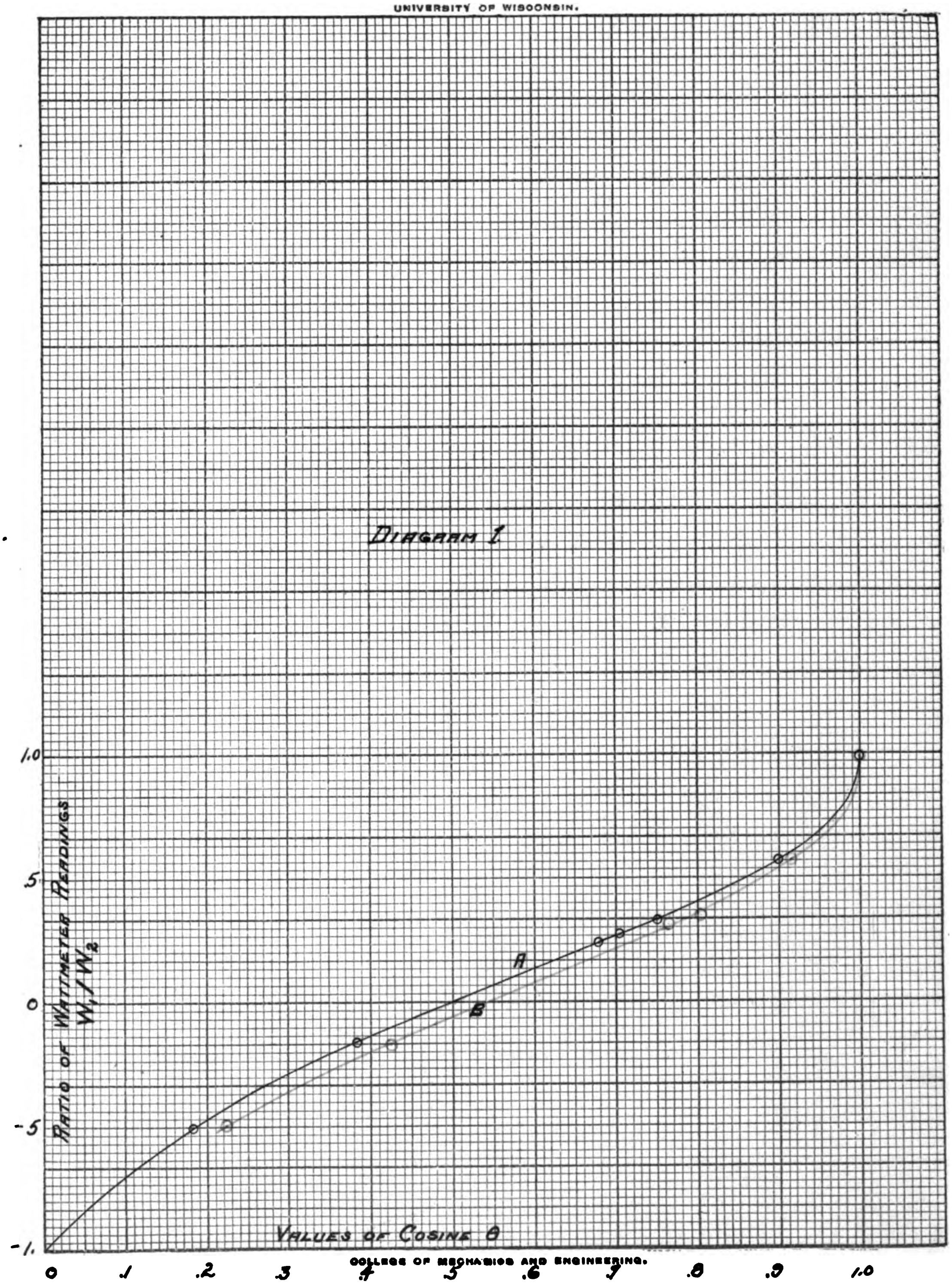
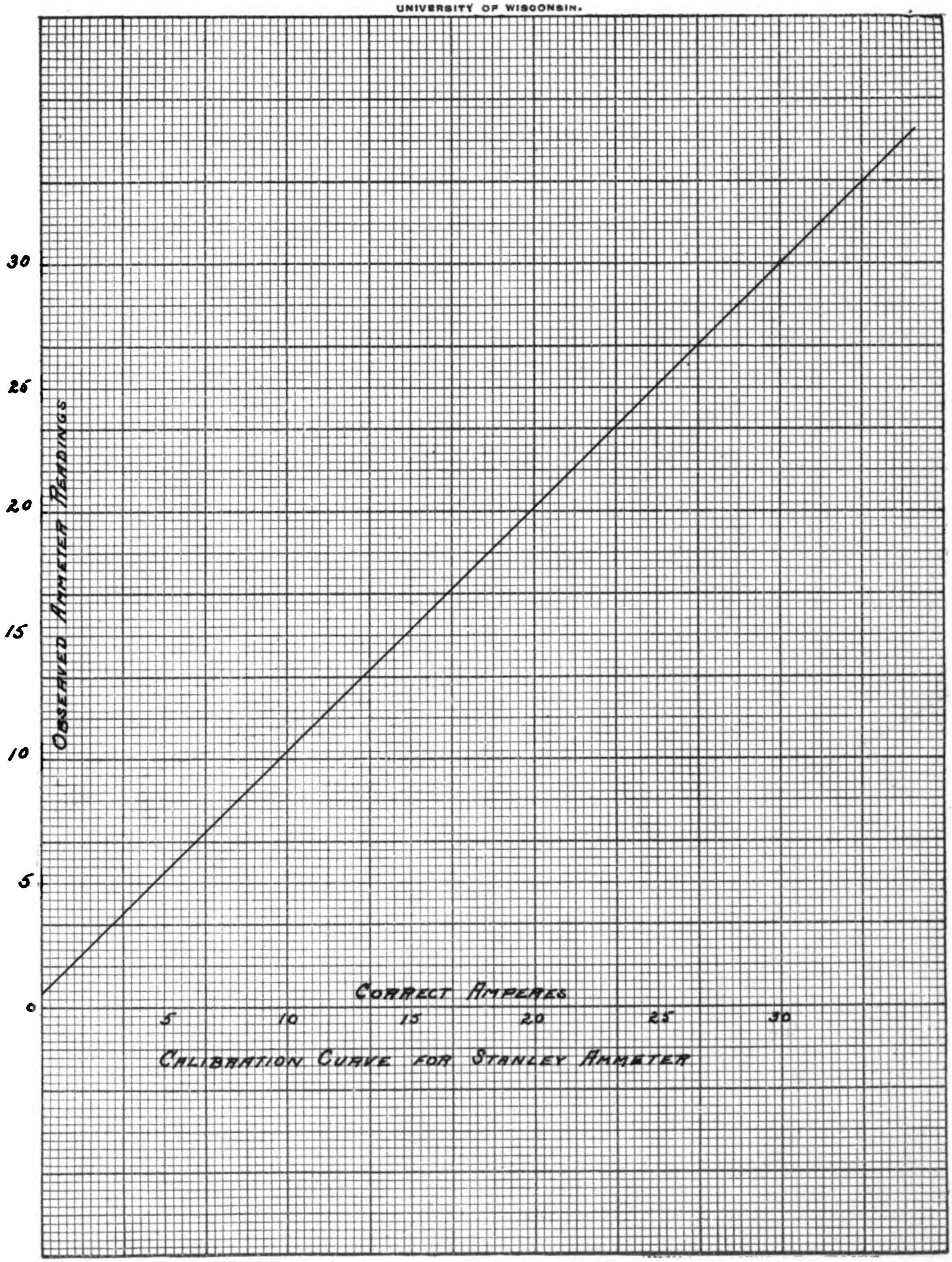
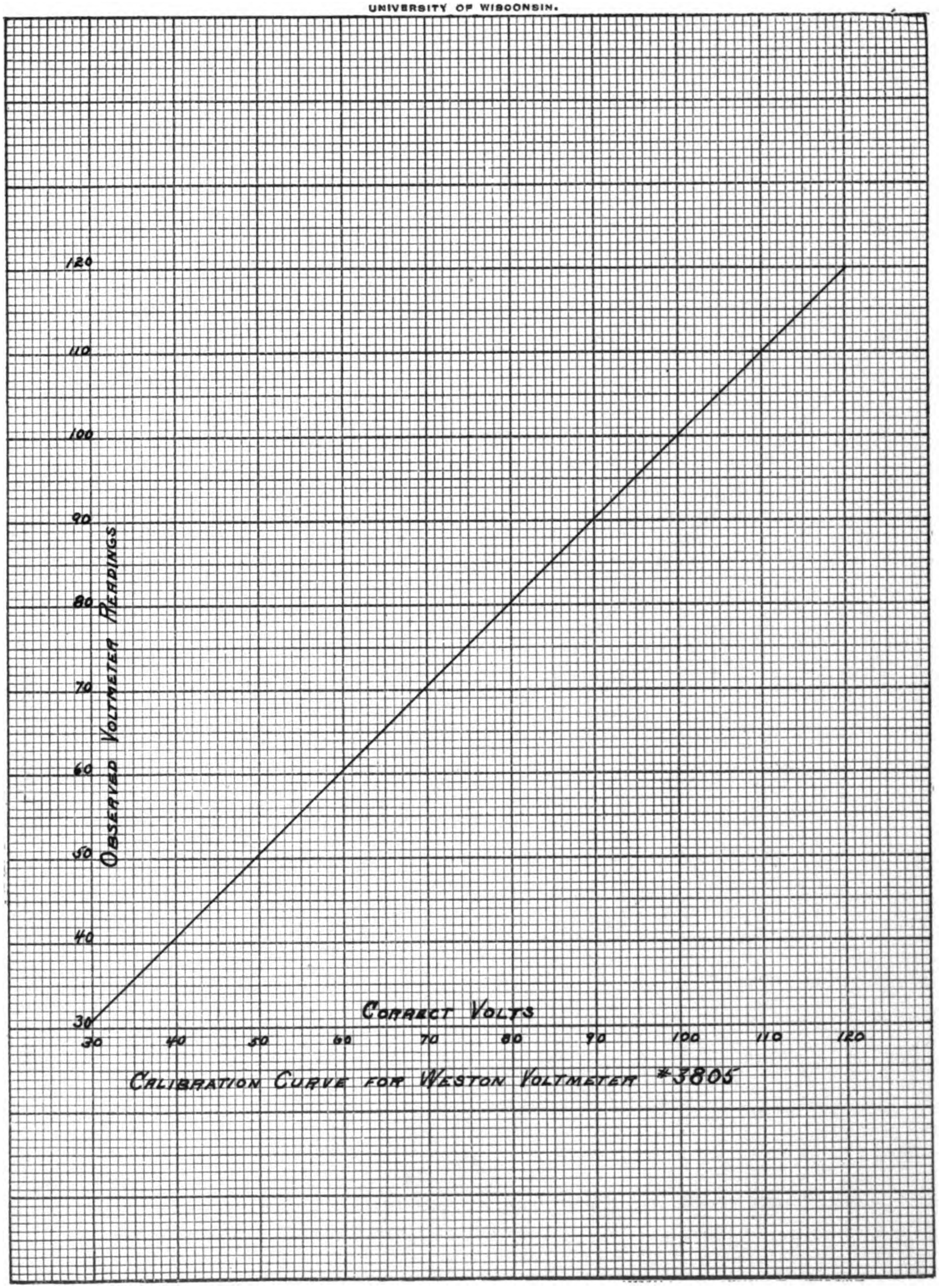


DIAGRAM I
1.0
.5
0
-.5
-1.
RATIO OF WATTMETER READINGS
W_1/W_2
A
B
VALUES OF COSINE θ
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0


30
25
20
15
10
5
0
Observed Ammeter Readings
Correct Amperes
5
10
15
20
25
30
Calibration Curve for Stanley Ammeter

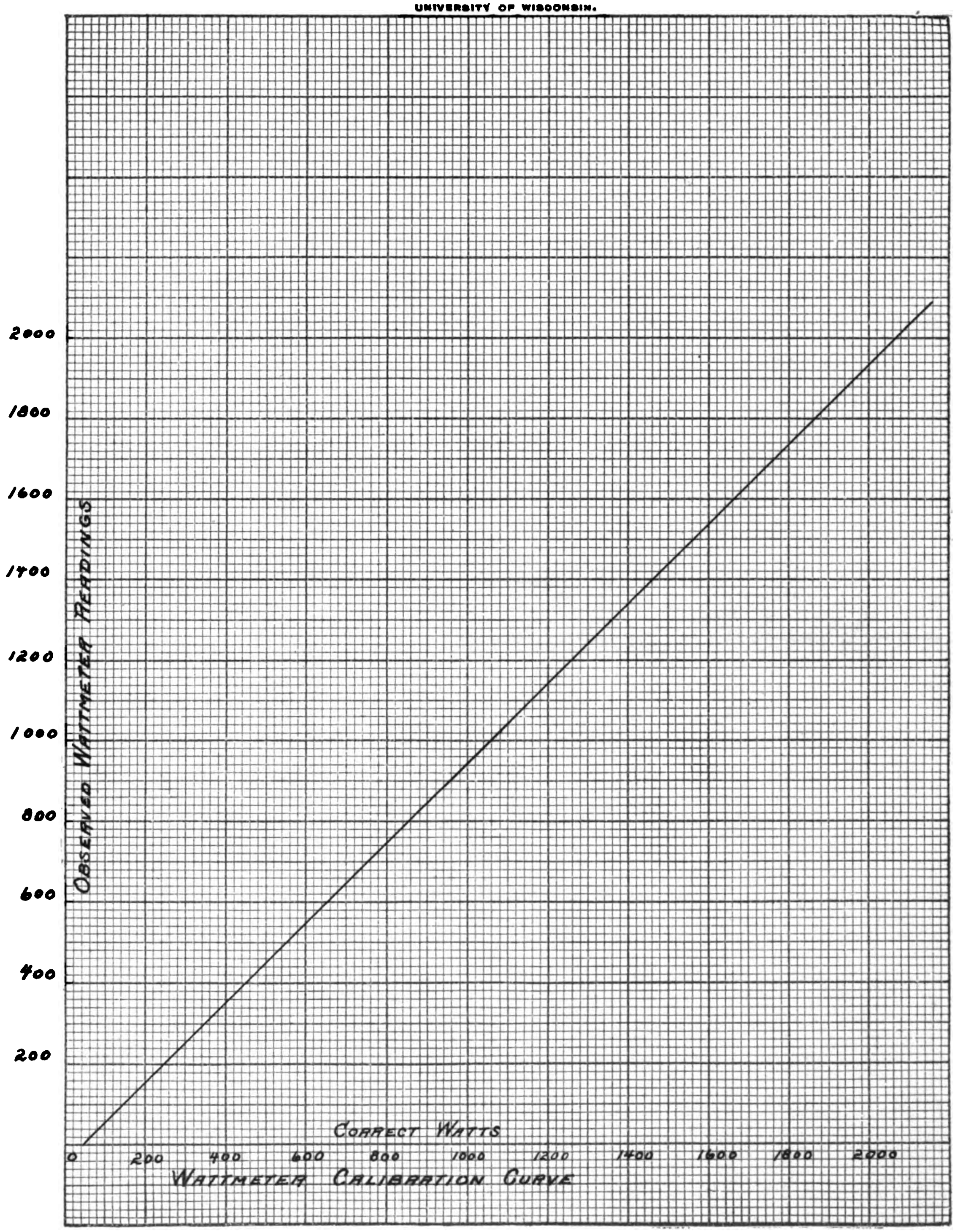
UNIVERSITY OF WISCONSIN.



Calibration Curve for Weston Voltmeter #3805

COLLEGE OF MECHANICS AND ENGINEERING.

UNIVERSITY OF WISCONSIN.



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APPROVED ....Geo. C. Shaad..........

..........Instructor in Electrical Engineering..........